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The Concept of 'Rate'

(more particularly, *angular* rate pertaining to rate gyroscopes)

Scope:

This applications note discusses in brief, the concept of rate*, in particular* angular rate*, as it pertains to gyroscopes and the measurement of angular velocity, or the rate-of-turn, along with how the data can be used. This applications note is primarily intended to familiarize the reader to the basic concepts relating to the rate gyroscope.*

What is *Rate*?

The word, *rate*, can refer to many different ideas. In engineering, rate refers to rate-of-change with respect to time. In mathematical language (operational calculus) rate is generally symbolized as:

$$\frac{dx}{dt}$$

which means the change in x with respect to the change in time *t* (the time during which the change in *x* occurred). '*x*' can symbolize real quantities such as position (distance), pressure, angle, energy, or even a rate (as in acceleration).

Typically in engineering, the above notation is taken to mean the *instantaneous* rate of change, or the time derivative. For example, the speedometer in a car indicates the instantaneous speed (rate of change of distance traveled) that the car is traveling at, at the *instant* during which one looks at the speedometer.

But another way to look at the change of a car's position with respect to time is as follows:

How long did it take to travel from point A to point B? If the distance was 4 miles, and it took 10 minutes to travel the 4 miles, then the average speed was:

$$\frac{4\ miles}{10\ minutes} \times \frac{60\ minutes}{1\ hour} = \frac{4\ miles}{1\ hour} \times \frac{6}{1} = \frac{24\ miles}{hour}$$

However, at any given time during which the car was traveling, it could have been moving at 50 mph, 10 mph, even 0 mph (stopped at an intersection), etc. This implies different speeds traveled at different times during the trip, but the total distance traveled over the total time of the trip is the average rate of speed.

Therefore, the rate of change of the car's position (from point A to point B) can be represented during the trip as a time derivative at any given time (instantaneously), or as an average rate at the *end* of the trip, when the total distance traveled and the time it took is tallied up and the math worked out.

It is extremely useful in many cases (if not *required*) in engineering to know what the given rate is at any given time, instantaneously, without having to wait until the event is over to do the accounting!

How is knowing a *Rate* useful?

We know that *rate* is a ratio of the quantity of change of anything, to the time up until the final point of the measurement. We can then use the ratio to determine what we want to know. For example, if the rate we are interested in is degrees of an angle traveled through per second (angular rate), and we keep track of the instantaneous rate of change and the time, then we can deduce certain information.

If the rate is 10° (or 10 of quantity *anything*) per second for one-half second, simple division shows the change in that half-second is 5°. If during the next half-second the rate increased to 15° per second, the change after one full-second is 5° + 7.5° = 12.5°. The ratio, or rate, is useful when the time during such change is accurately recorded, the total accumulated change (an increase in quantity) can be deduced.

Note: Accumulating a quantity over time is known as integration in operational calculus, and is the inverse of differentiation!

Therefore, the smaller one makes the time-slices for the purpose of the analysis, a more precise picture begins to appear. Operational calculus is designed to give that tool to engineering, where the time slices are considered to be infinitesimally small.

What about *Rate* Gyroscopes?

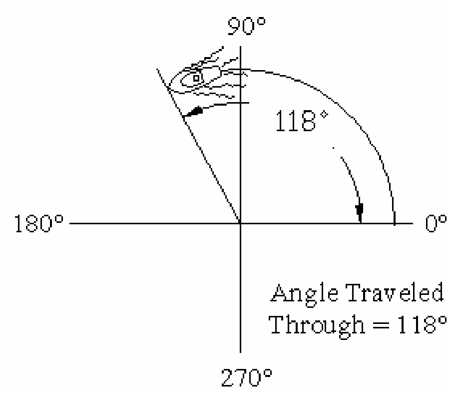
A rate gyroscope is a sensor that measures rates-of-turn, or *angular* rates (angular velocity), instantaneously. Typical units of measure are degrees per second. The rate gyroscope will not sense linear movements or accelerations. Spinning mass rate gyroscopes sense an input torque (angular motion only) presented at its sensitive (input) axis, and react with a precession torque from its output axis. The precession torque moves the gyroscope gimbal, which has attached to it an electrical signal generator.

An example of how a rate gyroscope could be used is as follows:

If a ship's pilot is told by the captain to quickly turn the ship around (a 180° about-face), and the ship manages to make the turn in 10 seconds, the average rate of the turn was then, 180°/10 = 18°/second.

However, suppose the ship's manufacturer warns that the rudder mechanism cannot take more than 22° per second during a maneuver. Then the pilot must make adjustments to the ship's speed *during* the maneuver to avoid damaging the ship's rudder. If he waits until the maneuver is complete and does the appropriate math to tell him that in-fact the ship made the turn of 180° in 10 seconds, he runs the risk of damage to the rudder if at *any* time during the maneuver the ship's rate of turn exceeds the 22° per second!

In this case, it is essential that the pilot has instantaneous rate-of-turn data available to him. A rate gyroscope is such an instrument designed to do this. Much like the speedometer in a car outputs the instantaneous rate of *linear* travel, the rate gyroscope outputs the instantaneous rate of turn (*angular* travel).



The figure above shows a ship in a 118° turn. The ship travels through the 118° arc in 10 seconds. Therefore, the ship's rate-of-turn or angular rate is 11.8° per second.

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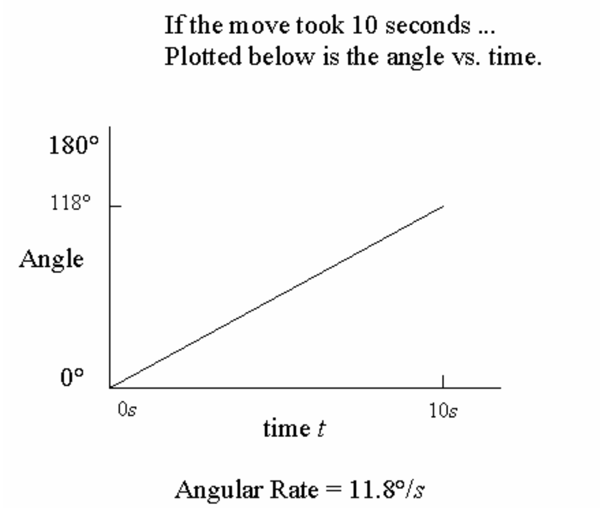
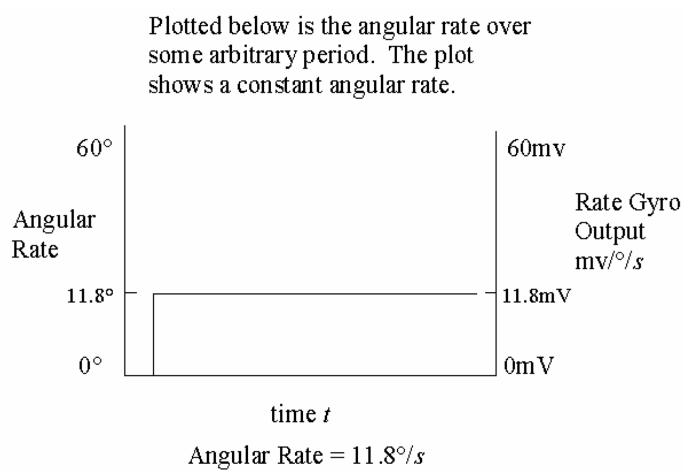


Above shows a plot of the turning *angle* over time. If the turn is made at a constant rate, the slope of the line will be linear. The slope of the line is 11.8°/sec, which is the rate of change of the angle over time.



Above is a plot of *angular rate* over time. It also shows the constant angular rate. The slope of the line is 0, which is the rate of change of the angular rate. Here, there is a constant angular rate. Also the typical rate gyro output in units of millivolts (mV) per degree per second is shown on the secondary vertical axis. Here, the rate gyroscope output *scale factor* is 1 mV/°/sec.

Other more common uses for the rate gyroscope are in platform stabilization, precision steering of antennas, and controlling the speed of rotating objects. The rate feedback from rate gyroscopes can be used as direct feedback input to a servo system.

For example, if a radar antenna is required to rotate at a constant precise rate, a rate gyroscope can be used as the rate-of-turn feedback device. Therefore, if a disturbance such as the wind tends to push the antenna so as to either aid or impede the rotation of the antenna, the instantaneous feedback of rate from the rate gyroscope can be fed into a servo amp, which in turn can either decrease or increase the energy input for the antenna rotation actuator.

Another use for the rate gyroscope is in platform stabilization. If a platform is required to be horizontally stable, and is supported on an axle, the axle must be free to rotate in order to provide corrections for horizontal errors. If a rate gyroscope is affixed to the platform such that its sensitive axis is parallel to the axis of rotation of the platform's supporting axle, then any rotation of the platform with respect to inertial space will be detected. If a known horizontal reference is established, then any error from this reference can be reported by the rate gyroscope, and corrective motion applied to the platform.

Usually, the rate of platform turn-from-horizontal is used as feedback for a corrective motion system comprising a servo amp and a motor or other actuator. Here, the correction is applied on-the-fly in real-time, such that the platform maintains its horizontal reference position continuously.

How can the *Rate* Gyroscope output be used?

Previously, an example of how the raw rate gyroscope data, which is simply the rate-of-turn or angular rate, could be used by a ship's pilot to monitor the turning rate of the ship much like a driver checks and manages the speed of a car. However, there is other information about angular quantities, such as angle and angular acceleration that can be extracted from the rate gyroscope's output.

If the rate gyroscope output data is fed into a system which can keep accurate track of time, then from that data one can determine how many degrees of arc has accumulated during the time of interest. This operation is called *integration*, where if a constant rate of turn is timed precisely, at any given time one can tell the exact degree of turn at any given time.

For example, for a constant rate gyroscope output representing 45° per second, over one and one-half seconds the turn is 45° + 22.5° = 67.5°.

If the output is not constant, an accurate computation can still be made if the time-slices used are small enough. Here, if for the first one-*quarter* second the rate was 20° per second, the second-*quarter* second, it was 40° per second, and the third-*quarter* second it was again 20° per second, the total degree of turn is; 5° + 10° + 5° = 20°. Of course the smaller the time slice, the more accurate the result.

Therefore, the raw output data from the rate gyroscope can tell the user what the instantaneous rate-of-turn is. If the data is integrated over some time, one can tell how many degrees of arc of the turn have occurred.

A third use of the rate gyroscope output, in addition to providing the actual instantaneous rate of turn, and the integrated rate, or the accumulated degree of turn, is that the angular acceleration can be deduced.

Angle	*Angular Rate*	*Angular Acceleration*
θ	$\dot{\theta}$	$\ddot{\theta}$

Dot Notation

Differentiation →

$$\theta \qquad \frac{d\theta}{dt} \qquad \frac{d^2\theta}{dt^2}$$

Derivative Notation

← *Integration*

$$\int_0^t \ddot{\theta}\, d^2t \qquad \int_0^t \ddot{\theta}\, dt \qquad \ddot{\theta}$$

Integral Notation

In the figure above, the operational calculus required to compute the angle traveled through, or the angular acceleration, from the rate gyroscope output is shown. If one *differentiates* the rate gyroscope output, the angular acceleration can be resolved. If one integrates the rate gyroscope output, the total accumulated angle traveled through can be resolved.

Summary:

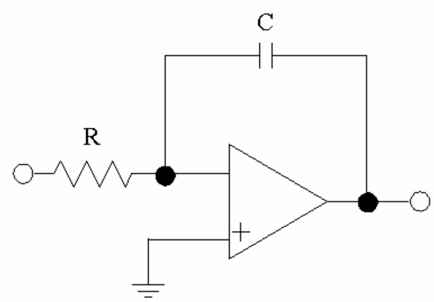
Using operational calculus,

Integrate the angular rate = the accumulated angle
Differentiate the angular rate = angular acceleration

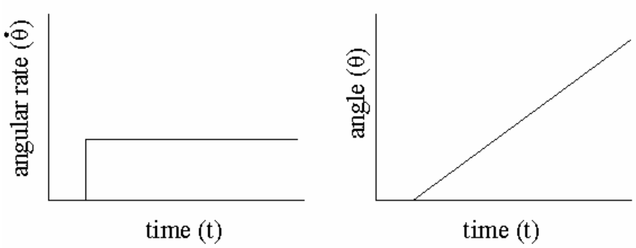
Can *Rate* Gyro output data be used, really?

Operational calculus has been shown to allow the manipulation of rate gyro output data to obtain other information, such as angle (accumulated) or angular acceleration. The same operational calculus can be easily implemented in modern electronics.

Typically, solid-state operational amplifiers, or op-amps, are used in various circuits to perform the data manipulation. Here, the electronics and associated methodology is said to be *analog*, as opposed to doing like manipulations in digital computers or more commonly today, DSP's or microcontrollers. For the purposes of the discussion herein, typical basic analog electronics will be described.



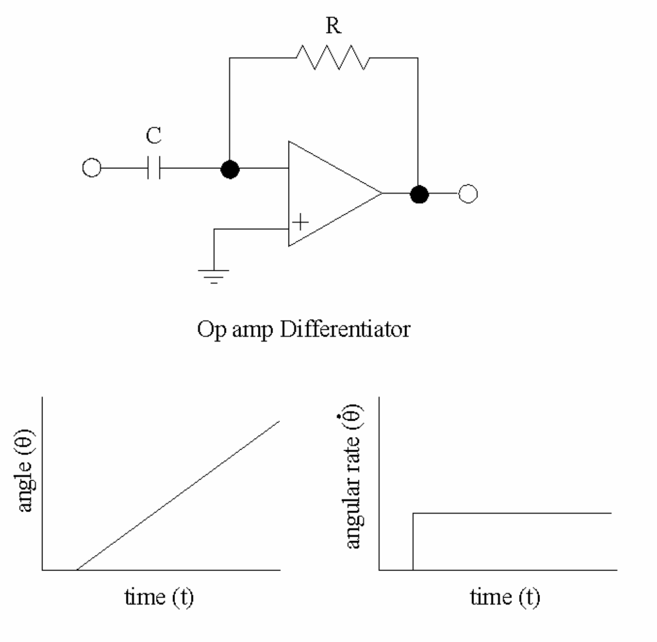
Op amp Integrator



The figure above shows an op-amp integrator. The graph on the left shows an input of a constant angular rate (or the amount of rate-of-change). The graph on the right shows an output from the integrator. The sloped line represents the *integrated* (accumulating) angle turned through.

Here, in real-time, the capacitor (C) charges between the resistor (R) and the output of the amplifier. Realistically however, if the constant rate continues indefinitely, the capacitor will charge to the level of the power supply from which the op-amp is powered. Note that real-time infers no time-slices. The process is continuous.

The sloped line on the graph on the right represents the *rate* of capacitor charging! Not to be confusing, but the sloped line is called the time-derivative of the angle traveled through! That is, for any time value picked on the horizontal axis of the graph, the corresponding point on the vertical axis is the exact angle at that precise instant in time. Here, the reciprocal (*inverse*) property of the operational calculus is evident.



The figure above shows a basic op-amp differentiator. The graph on the left shows a constant accumulation of an angle traveled through, or of an angular rate. The graph on the right shows a differentiation of that accumulated angle, or the 'level' of the accumulating angle or angular rate.

In an op-amp differentiator like the one shown above, the input capacitor can only react to a change in input signal.

Note: Capacitors block direct current DC, and pass alternating (changing) AC current.

Therefore, the sloped line in the graph on the left representing the derivative of the angle at any given instant is converted to a constant level shown in the graph on the right. This means that the capacitor charges and discharges into the op-amp at a constant rate. The graph on the right is then the level of the charge/discharge cycle occurring continuously (as long as the rate of change of the input is linear [constant]), or the *amount* of the rate-of-change.

Of further use for the output of the rate gyroscope, one can differentiate the rate-of-change of a rate-of-turn to arrive at angular acceleration! The graphs would look like those in the op-amp differentiator figure above. The graph on the left however, would be labeled angular rate on the vertical, and the graph on the right, angular acceleration on the vertical.

Note: A constant acceleration characteristic will give an ever increasing velocity. The acceleration when integrated will give velocity! (See the graphs for the op-amp integrator.)

Using op-amps in analog circuitry, one can manipulate rate-of-turn data from a rate gyroscope to determine total accumulated angle traveled through by integration or the angular acceleration by differentiation. The rate gyroscope then becomes useful as a protractor (for determining angle) and an angular accelerometer for determining the angular acceleration.

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Note: Performing practical operational manipulation of data requires an accurate time-base, accurate low-drift electronic components, and a minimized drift characteristic of the rate gyroscope.

Closure…

The above discussions explored the concept of *rate*. More specifically focused on was the concept of rate-of-turn, and the rate gyroscope as the sensing instrument. Rate gyroscopes sense angular motions only.

Rate information is useful in many situations, from determining if the local highway speed limit is being exceeded, to determining location during navigation.

Rate information can also be extended by performing operational calculus to obtain absolute position, or acceleration. Performing the operational calculus requires accuracy in the root measurement as well as the time-base used. In the case of the op-amp circuits, low leakage capacitors, drift-free amplifiers, low temperature coefficient resistors, etc., are required. If digital technology is used, an accurate timing frequency for the processor and careful implementation of the software code is required.